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An analysis of the steady state thermal behaviour of electrolysers with recycle

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A mathematical model, based on thermal balances and heat transport equations for the computation of steady state values of key temperatures and heat loss rates, is employed to analyse electrolyser performance. In particular, the effect of recycling rate and thermal insulation thickness are investigated from the viewpoint of cost-optimal operation.

Notation

A_E heat transfer loss area (electrolyser)
A_T heat transfer loss area (recycle line)
c_H specific cost of thermal energy
c_i specific cost of insulation (including labour and amortisation)
c specific heat capacity of electrolyte
d_i thickness of lagging on recycle line
F Faraday constant
f_R recycle ratio (Q_R/Q)
H height of electrolyser
H_E rate of heat loss from electrolyser
H_T rate of heat loss from recycle line
h_0 outer-surface heat transfer coefficient
I electric current
k_i thermal conductivity of lagging
L_E length of electrolyser
L_R length of the recycle line
MU symbol of an arbitrary monetary unit
Q volumetric flow rate of inlet electrolyte
Q_R volumetric flow rate of recycle stream
R_E electrolyte resistance in electrolyser
r_i inner radius of recycle line
T_A ambient temperature
T_m temperature of electrolyte entering the electrolyser
T_R temperature of electrolyte leaving the recycle line
T_1 temperature of fresh intake electrolyte
T_2 temperature of electrolyte leaving the electrolyser
t_i thickness of insulation on electrolyser
U_E overall heat transfer coefficient (electrolyser)
U_T overall heat transfer coefficient (recycle line)
V_i volume of lagging
W width of electrolyser
z valency
ΔH_R heat of reaction
ΔT_{LM} log mean temperature (electrolyser)
$\Delta\Theta_{LM}$ log mean temperature (recycle line)
Θ symbol for time
ρ density of electrolyte

1. Introduction

The beneficial effect of recycling on electrolyser performance has been well documented in the electrochemical engineering literature [1–3]; recycling is particularly attractive when single-pass active ion-to-product conversions are low in order to increase the efficiency of electrolysis. Various design strategies for specified performance criteria have also been described [4]. In these studies isothermal conditions were *ab ovo* assumed for the sake of simplicity, but isothermality depends on a careful matching of heat generation (for example, *via* Joule heat) and heat loss rates, hence an *a priori* assumption of constant temperature in the electrolyser, and in the recycle tube is not always justified. On the other hand, it may be desirable to maintain at least near-isothermal conditions in a given electrolyser/recycle system in order to suppress, for example, parasitic chemical reactions which may occur at a significant rate within a certain temperature range. Alternatively, it may be important to maintain a particular temperature profile or level in the system on account of a specific technical or economic reason. The purpose of this paper is to present an approximate mathematical framework for a rapid analysis of thermal behaviour of such systems and to show via a numerical illustration the possibility of thermal control via a judicious choice of heat loss parameters and the recycle rate. The results may be of direct interest to the rational design of industrial-scale electrolyser systems where electrolyte recycling may be one of several possible choices, out of technical and ecological considerations.

2. Model development

The electrolyser-with-recycle system is sketched in Fig. 1. In describing its thermal behaviour the follow-

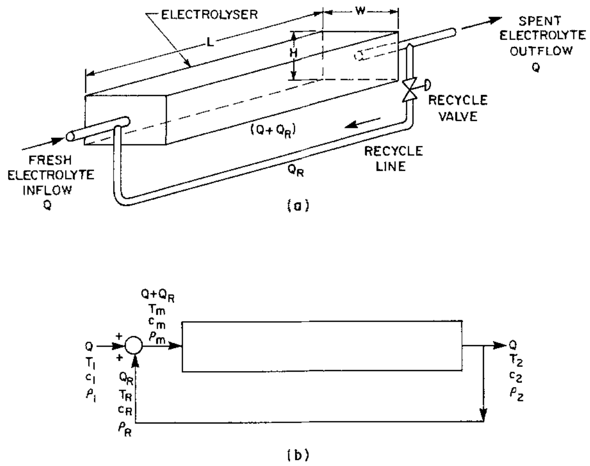

Fig. 1. Sketch of an electrolyser with recycle. (a) Physical layout, (b) block diagram and notation.

ing assumptions are made:

1. The system operates in the absence of dynamic perturbations, hence the governing thermal balances are time-invariant. The resulting model allows the estimation of steady-state conditions as a function of process variables and parameters. Transition between various steady-state levels due to perturbations in such variables and/or parameters is beyond the scope of this paper.

2. In order to simplify computational encumbrance, the temperature dependence of the electrolyte specific heat capacity, density and electric conductivity is taken into account in terms of these quantities averaged over the temperature range of interest. Simlarly, the enthalpy change due to the overall electrochemical reaction (heat of reaction) is considered to depend only slightly on temperature hence it can also be considered as an approximately averaged value.

3. Thermal conduction in the direction of electrolyte flow is negligible with respect to convective heat transport by the electrolyte, and heat transport normal to the flow of the electrolyte.

4. The thermal resistance to heat flow through the metallic tube wall in the recycle section and the electrolyser tank material is negligible with respect to all other thermal resistances (the thermal conductivity of conventional heat exchanger metals e.g. copper is at least two orders of magnitude higher than the thermal conductivity of conventional thermal lagging materials).

There is much evidence in the heat transfer literature and in industrial practice for the engineering soundness of assumptions 2–4 under normal operating conditions. The model developed in the sequel is robust in the sense that slight to moderate changes in pertinent operating variables and parameters do not predict extraordinary thermal behaviour, due to the absence of mathematical singularities in it. The basic "building blocks" of the model are three thermal energy transfer rate equations, or balances:

(i) the thermal balance over the mixed-inlet section:

$$Qc\rho T_1 + Q_R c\rho T_R = Q(1 + f_R)c\rho T_m \qquad (1)$$

(ii) the thermal balance over the electrolyser section (neglecting losses over parasitic resistances, for example, couplings and contact surfaces):

$$Q(1 + f_R)c\rho T_2 = Q(1 + f_R)c\rho T_m + I^2 R_E - \frac{\Delta H_R I}{zF} - U_E A_B \Delta T_{LM} \qquad (2)$$

(iii) the thermal balance over the recycle section:

$$Q_R c\rho T_2 = Q_R c\rho T_R + U_T A_T \Delta\Theta_{LM} \qquad (3)$$

In this algebraic structure the fresh inlet electrolyte temperature (T_1), the inlet electrolyte flow rate (Q), the recycle ratio (f_R), and the electric current (I) are the input variables for a given size of the electrolyser, the recycle line, and the thermal insulation thickness. The temperature of the electrolyte leaving the electrolyser (T_2), entering the electrolyser (T_m), and leaving the recycle line (T_R) are the output variables. Since they are interlinked via the log mean temperatures ΔT_{LM} and $\Delta\Theta_{LM}$, an iterative solution scheme is necessary even if all heat transport parameters were considered to be constant including the overall heat transfer coefficients (U_E, U_T) whose dependence on system geometry and insulation size is demonstrated in the sequel. The physical parameters are appropriately taken mean values as indicated above. An alternative formulation of balance (iii), based on a differential thermal balance over the recycle section (as shown in the Appendix) yields the explicit relationship

$$T_R = T_A + (T_2 - T_A)\exp\left(-\frac{U_T A_T}{Q_R \rho c}\right) \qquad (4)$$

and removes the necessity of iterations over Equation 3.

3. Estimation of heat-loss characteristics

In the absence of thermal insulation, the rate of heat transport through the electrolyser walls and the recycle tube is determined primarily by natural convection from the outer surfaces into the surroundings. Thermal insulation decreases the rate; the higher its thickness, the lower the overall heat transfer coefficient. Various formulae in the literature, for example [5, 6], for the estimation of the natural convective heat transfer coefficients indicate that the value of $h_o = 5\,\mathrm{W\,m^{-2}K^{-1}}$ for the uninsulated electrolyser and $h_o = 6\,\mathrm{W\,m^{-2}K^{-1}}$ for the uninsulated recycle pipe may be taken; in the case of conventional insulation, $h_o = 3\,\mathrm{W\,m^{-2}K^{-1}}$ [7] is recommended at still ambient temperatures about 20° C. If there is considerable air movement, the value of h_o may rise to as high as $35\,\mathrm{W\,m^{-2}K^{-1}}$ (ave. air velocity of $\sim 32\,\mathrm{km\,h^{-1}}$) in the case of a 2.54 cm (1 in nominal) uninsulated pipe [5]. If the recycle pipe is insulated with a lagging of d_i thickness, the overall heat transfer coefficient is computed as [7]

$$\frac{1}{U_T} = \frac{r_i + d_i}{k_i}\ln\frac{r_i + d_i}{r_i} + \frac{1}{h_o} \qquad (5)$$

since the thermal resistance of the metallic pipe wall may be neglected. It follows that the rate of heat loss from the pipe reaches a maximum of the critical insulation thickness

$$d_i = \frac{k_i}{h_o} - r_i \tag{6}$$

indicating that at overall radii less than k_i/h_o, the addition of lagging will increase, and not decrease, heat losses on account of increased surface areas, a well-known phenomenon in heat transport theory. Equation 5 also implies that resistance to heat transfer from the electrolyte to the recycle pipe wall is much smaller than the outer resistance. Heat losses from the electrolyser may be estimated in a similar manner. Assuming a negligible internal resistance to heat flow and no heat transport through the bottom of the electrolyser into the floor, the overall loss area (see Fig. 1) may be expressed as

$$A_E = (2HL_E + 2WH + WL_E) + 4(L_E + H)t_i \tag{7}$$

The overall heat transfer coefficient is given by

$$\frac{i}{U_E} = \frac{t_i}{k_i} + \frac{1}{h_o} \tag{8}$$

In contrast with an insulated pipe, the rate of heat transport per unit temperature gradient decreases monotonically with the thickness of insulation.

As seen in Equations 5 and 8, the magnitude of the thermal conductivity of the lagging is a predominant factor in determining the overall rate of heat loss: it varies typically from $0.04\,\mathrm{W\,m\,K^{-1}}$ (blanket of batt insulation: mineral wool) to $0.72\,\mathrm{W\,m^{-1}\,K^{-1}}$ (common brick). In the case of asbestos ($k = 0.17\,\mathrm{W\,m^{-1}\,K^{-1}}$); [7] a one cm thick lagging on a 2.54 cm (nominal size 1 in) bare pipe reduces the overall heat transfer coefficient by about 18%; on a bare vertical surface the reduction is about 15%.

4. Computation of thermal behaviour

In galvanostatic (constant current) electrolysis, if the mean value of the electrolyte resistance taken over the temperature change in the electrolyser is R_E, the lumped parameters

$$a \equiv \frac{I^2 R_E - \Delta H_R I/zF}{(1 + f_R)Q\rho c} \tag{9}$$

$$b \equiv \frac{U_E A_E}{(1 + f_R)Q\rho c} \tag{10}$$

may be considered as constants, under a given set of operating conditions.

Then, three explicit relationships emanating from the thermal balances:

$$T_2 = T_m + a - b\Delta T_{LM} \tag{11}$$

$$T_R = T_A + (T_2 - T_A)\exp\left(-\frac{U_T A_T}{Q_R \rho c}\right) \tag{12}$$

$$T_m = T_1 \frac{1 + (T_R/T_1)f_R}{(1 + f_R)} \tag{13}$$

are interlocked via the logarithmic mean temperature drop

$$\Delta T_{LM} = \frac{(T_2 - T_m)}{\ln\left\{\dfrac{T_2 - T_A}{T_m - T_A}\right\}} \tag{14}$$

requiring an iterative solution where T_2, T_R and T_m are *a priori* unknown while all operation parameters and input variables are specified numerically. If a conventional Gauss-Seidel iteration procedure is employed, a value of T_2 is assumed first, T_R is computed via Equation 12, T_m is computed via Equation 13 and ΔT_{LM} is computed via Equation 14. Then, an improved value of T_2 is computed via Equation 11. The procedure is stopped when no further improvement in the numerical value of T_2 is found. The various components of the heat balance can then be evaluated in a straightforward manner. It may be shown without undue difficulty that the system of Equations 11–14 obeys the contraction mapping theorem [8]; the rate of convergence of the Gauss-Seidel iteration is similar to that of the bisection method where $\ln 100a/\ln 2$ iterations are necessary [9] if the starting interval is a units wide, for a 0.01 magnitude error in the estimated value of T_2 (with $a = 5°\mathrm{C}$, the number of iterations is nine).

5. Effect of economic factors on process performance

As shown in section 3 heat losses from the recycle line are the highest at the critical insulation thickness, where the overall radius of the insulated recycle pipe equals the ratio of the thermal conductivity of the lagging to the heat transfer coefficient at the bare outer pipe surface. These conditions are not necessarily the worst, however, from an economic point of view, if the cost of the unrecovered heat and insulation are simultaneously considered (the unrecovered heat, i.e. heat lost to the surroundings is considered as an "exergic debit", hence a financial loss). The total cost of heat and insulation

$$C_T = c_i V_i + c_H(H_E + H_T) \tag{15}$$

is a complicated function of the lagging thickness, especially if both electrolyser tank and recycle tube are insulated. In the simpler case where only the recycle tube is insulated, H_T reaches a maximum at the critical value of $d_i = k_i/h_o$, and H_E reaches a minimum where ΔT_{LM} reaches a minimum. However, the total cost is the highest when $d_i = 0$ and reaches a local minimum at an intermediate d_i value, determined by the ratio of specific costs. As d_i is increased, the total cost rises with the thickness of recycle-line insulation towards an asymptotic value. The extremal values of C_T may be computed, in principle, via Equation 16 (derived directly from Equation 15):

$$\frac{dV_i}{d(d_i)} = -\frac{c_H}{c_i}\frac{d}{d(d_i)}(H_E + H_T) \tag{16}$$

but it is obvious by inspection that if c_H/c_i is large, C_T reaches its minimum at a particular value of d_i where

Table 1. Numerical values of process parameters in the illustrative example

Parameter and symbol	*Numerical Value*	*Unit*
Electrolyser width, W	0.1	m
Electrolyser height, H	0.1	m
Electrolyte density, ρ	1015	$kg\,m^3$
Electrolyte specific heat capacity, c	4187	$J\,kg^{-1}K^{-1}$
Overall heat of reaction in electrolyser, ΔH_R	−200	$kJ\,mol^{-1}$
Electrolyte flow rate, Q	5×10^{-7}	m^3s^{-1}
Valency, z	2	
Inlet temperature of fresh electrolyte, T_1	22	°C
Ambient temperature, T_A	20	°C

$(H_E + H_T)$ is decreasing but its d_i derivative is sufficiently close to zero. In practical terms, C_T and $(H_E + H_T)$ reach their minima at essentially the same value of d_i. It is worth noting that although C_T does *not* represent the full cost of recycle operation, it is independent of pumping costs of recycling, since the latter is usually expressible as a simple power function of the volumetric flow rate [10].

6. Effect of process variables on performance: a numerical illustration

Table 1 summarizes the constant parameters of a small scale electrolytic process employed for the illustration of process performance. The active electrode area is $25\,cm^2$ for ever cm of active electrode length, and the design conversion without recycle is 50% (one half of the active ionic component is converted into an electrolysis product per single pass). In Table 2 conversion, as a figure merit of recycling, and as a function of pertinent parameters is given: the computation has been discussed earlier (4). The recycle tube is a 2.54 cm (1 in Schedule 40) pipe of total active length of 1.9 m, insulated with asbestos lagging of thermal conductivity $k_i = 0.17\,W\,m^{-1}K^{-1}$ [7]. The h_o values have been assigned according to Section 3.

In Tables 3 and 4 the key temperatures and heat transfer rates are assembled at two levels of electric

Table 2. Process parameters determined by the recycle ratio (f_R).

Parameter	$f_R = 1$	5	10	15
Length of electrolyser, L_E(m)	1.9	1.9	1.9	1.9
Recycle flow rate Q_R, (m^3s^{-1})	5×10^{-7}	2.5×10^{-6}	5×10^{-6}	7.5×10^{-6}
Conversion with recycle (%)	52.3	58.3	62.3	64.8

Table 3. The effect of the recycle ratio on thermal performance for $I = 25\,A$; $R_E = 0.1\,\Omega$; $t_i = 1\,cm$.

	d_i (*cm*)	T_2 (°*C*)	T_R (°*C*)	T_m (°*C*)	H_i^* (*W*)	$H_o^\dagger$ (*W*)	H_E (*W*)	H_T (*W*)
$f_R = 1$	0	44.52	38.54	30.27	128.64	189.21	27.82	12.72
	1	43.91	36.57	29.29	124.46	186.60	26.29	15.58
	2	43.64	35.73	28.86	122.67	185.45	25.59	16.80
	3	43.54	35.41	28.71	121.99	185.03	25.34	17.27
	4	43.52	35.34	28.67	121.84	184.94	25.28	17.38
	5	43.53	35.38	28.69	121.93	185.00	25.31	17.31
	6	43.56	35.49	28.74	122.16	185.13	25.40	17.16
	7	43.60	35.62	28.81	122.44	185.31	25.50	16.96
$f_R = 5$	0	41.88	40.69	37.58	479.07	533.97	33.39	12.66
	1	41.09	39.60	36.67	467.47	523.87	31.92	15.83
	2	40.73	39.11	36.26	462.25	519.28	31.26	17.23
	3	40.59	38.92	36.10	460.27	517.55	31.01	17.77
	4	40.56	38.88	36.07	459.81	517.15	30.96	17.90
	5	40.58	38.91	36.09	460.10	517.40	30.99	17.82
	6	40.63	38.97	36.14	460.75	517.97	31.08	17.64
	7	40.68	39.04	36.20	461.58	518.69	31.18	17.41
$f_R = 10$	0	41.42	40.83	39.11	914.25	968.06	34.39	12.56
	1	40.59	39.85	38.23	893.51	948.75	32.93	15.74
	2	40.23	39.43	37.84	884.50	940.41	32.30	17.16
	3	40.09	39.26	37.69	881.01	937.17	32.06	17.71
	4	40.06	39.22	37.66	880.20	936.42	32.00	17.84
	5	40.08	39.25	37.68	880.71	936.89	32.03	17.76
	6	40.13	39.30	37.73	881.87	937.96	32.11	17.57
	7	40.19	39.37	37.79	883.33	939.32	32.22	17.34

* $H_i = Q(1 + f_R)c\rho T_m$
† $H_o = Q(1 + f_R)c\rho T_2$

Table 4. The effect of the recycle ratio on thermal performance for $I = 30\,A$; $R_E = 0.09\,\Omega$; $t_i = 1\,cm$.

	d_i (cm)	T_2 (°C)	T_R (°C)	T_m (°C)	H_i (W)	H_o (W)	H_E (W)	H_T (W)
$f_R = 1$	0	50.87	43.33	32.67	138.83	216.19	34.72	16.01
	1	50.10	40.86	31.43	133.58	212.90	32.76	19.62
	2	49.77	39.81	30.91	131.34	211.51	31.91	21.16
	3	49.65	39.41	30.70	130.49	210.98	31.59	21.75
	4	49.62	39.32	30.66	130.29	210.86	31.52	21.89
	5	49.64	39.37	30.69	130.41	210.94	31.56	21.80
	6	49.68	39.51	30.75	130.70	211.11	31.67	21.61
	7	49.73	39.67	30.84	131.05	211.33	31.80	21.36
$f_R = 5$	0	47.50	46.00	42.00	535.52	605.61	41.88	15.90
	1	46.50	44.63	40.86	520.92	592.89	40.04	19.90
	2	46.05	44.01	40.35	514.38	587.14	39.21	21.65
	3	45.88	43.78	40.15	511.89	584.97	38.90	22.34
	4	45.84	43.73	40.10	511.31	584.47	38.83	22.49
	5	45.87	43.76	40.13	511.67	584.78	38.87	22.39
	6	45.92	43.84	40.20	512.50	585.50	38.98	22.17
	7	46.00	43.94	40.28	513.54	586.41	39.11	21.88
$f_R = 10$	0	46.91	46.17	43.97	1027.80	1096.52	43.18	15.78
	1	45.88	44.95	42.86	1001.79	1072.31	41.35	19.78
	2	45.43	44.41	42.37	990.44	1061.78	40.55	21.56
	3	45.25	44.20	42.19	986.04	1057.70	40.24	22.25
	4	45.21	44.16	42.14	985.02	1056.76	40.17	22.41
	5	45.24	44.19	42.17	985.66	1057.35	40.21	22.31
	6	45.29	44.25	42.23	987.12	1058.70	40.32	22.08
	7	45.37	44.34	42.31	988.96	1064.41	40.45	21.80

current and at a fixed insulation thickness for the electrolyser. At a fixed Joule heat ($I^2 R_E$) and recycle ratio (f_R), the temperatures and the heat transfer rates reach local extrema at the critical insulation thickness (about 4 cm). Temperature levels rise as $I^2 R_E$ is increased and it follows that if $I^2 R_E$ is large, partial electrolyte evaporation would have to be taken into account in computing heat transport and conversion rates due to a strong increase in all temperatures and heat transfer. However, if R_E is sufficiently low, the effect of insulation thickness on thermal behaviour becomes minimal and near isothermal conditions in the overall system can be maintained. An increase in the thickness of lagging below the critical value will increase the ($T_2 - T_R$) drop, but past the critical value of d_i ($T_2 - T_R$) reaches an essentially constant value. When f_R is increased, the ($T_2 - T_R$) drop decreases and the effect of the lagging thickness on it becomes minimal.

The effect of the electrolyser insulation thickness is portrayed in Tables 5 and 6. In the case of uninsulated electrolyser and recycle line the key temperatures are significantly lower than in the case of insulation, and at sufficiently high recycling rates, the system is near-isothermal. Unrecovered heat losses are the highest with respect to figures shown in Tables 3, 4 and 6. The insulation effect is further illustrated in Fig. 2, where the variation of the total thermal cost with recycle pipe insulation thickness is plotted at various operating conditions. The cost of unrecovered heat reaches a high-d_i asymptote after passing through a local minimum ($d_i = 1$ cm); the higher the recycle ratio the higher the thermal cost. The cost-optimal insulation thickness on the recycle-line is considerably smaller than the value ($d_i \simeq 5$ cm) associated with the largest

Table 5. Thermal behaviour in the absence of all insulation ($t_i = 0$; $d_i = 0$)

f_R	T_2 (°C)	T_R (°C)	T_M (°C)	H_i (W)	H_o (W)	H_E (W)	H_T (W)	C_H (MU s^{-1})
1	39.20	30.97	26.49	112.56	166.60	34.37	17.49	57.77
5	35.89	34.21	32.17	410.21	457.61	40.93	17.88	64.16
10	35.35	34.52	33.38	780.19	826.32	42.07	17.76	66.65

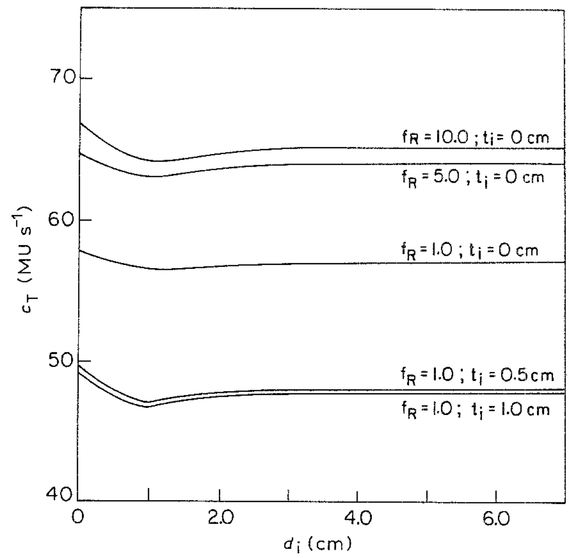

Fig. 2. The effect of insulation thickness on the total heat cost of the electrolyser-with-recycle system.

Table 6. The effect of electrolyser insulation on thermal performance for $I = 25\,A$; $R_E = 0.1\Omega$; $f_R = 1$.

	d_i (cm)	T_2 (°C)	T_R (°C)	T_m (°C)	H_i^* (W)	$H_o^\dagger$ (W)	H_E (W)	H_T (W)
$t_i = 2$	0	42.97	33.12	27.56	117.30	182.60	22.93	20.92
	1	44.09	36.70	29.35	124.73	187.38	25.76	15.70
	2	43.82	35.85	28.93	122.93	186.24	25.10	16.94
	3	43.71	35.52	28.72	122.23	185.77	24.83	17.40
	4	43.69	35.45	28.72	122.07	185.67	24.78	17.51
	5	43.70	35.50	28.75	122.17	185.73	24.81	17.44
	6	43.75	35.61	28.80	122.41	185.91	24.91	17.29
	7	43.79	35.74	28.87	122.70	186.09	25.01	17.09
$t_i = 3$	0	43.09	33.19	27.60	117.28	183.11	22.56	21.03
	1	44.23	36.80	29.40	124.94	187.99	25.36	15.80
	2	43.96	35.94	28.97	123.13	186.83	24.70	17.03
	3	43.86	35.62	28.81	122.43	186.39	24.45	17.50
	4	43.83	35.54	28.77	122.28	186.28	24.40	17.61
	5	43.85	35.59	28.80	122.38	186.35	24.43	17.54
	6	43.88	35.70	28.85	122.60	186.49	24.51	17.39
	7	43.93	35.83	28.92	122.89	186.68	24.62	17.19
$t_u = 5$	0	43.26	33.29	27.65	117.49	183.86	22.02	21.19
	1	44.44	36.94	29.47	125.25	188.87	24.78	15.93
	2	44.16	36.08	29.04	123.41	187.68	24.13	17.18
	3	44.06	35.75	28.87	122.71	187.23	23.89	17.65
	4	44.03	35.67	28.84	122.55	187.12	23.83	17.76
	5	44.05	35.72	28.86	122.65	187.19	23.87	17.69
	6	44.08	35.83	28.91	122.88	187.34	23.95	17.53
	7	44.13	35.97	28.98	123.17	187.53	24.05	17.34

* $H_i = Q(1 + f_R)c\rho T_m$

† $H_o = Q(1 + f_R)c\rho T_2$

rate of heat loss from the pipe. Figure 2 also indicates that the thermal costs show the largest sensitivity to the thickness of electrolyser insulation at low values of t_i, and reductions in C_T become progressively smaller as t_i exceeds the value of 0.5. The numerical analysis of the illustrative example yields, therefore, the cost-optimal insulation thickness values of $d_i = 1$ cm; $t_i = 0.5$ cm for a 1:1 recycle ratio. Similar calculations at arbitrary recycle ratios are omitted inasmuch as they do not offer qualitatively different results.

7. Conclusions

The foregoing model of thermal behaviour is recommended for the design of steady state temperatures of the electrolyte exit stream, mixed inlet stream and the recycle exit stream in an electrolyser equipped with a recycle line. It is particularly useful in analysing the combined effect of recycle rate and the thickness of thermal insulation applied to the electrolyser, or the recycle pipe, or both. It demonstrates that the superficial "the thicker the insulation the better" assumption is incorrect, as indicated by heat transport theory. Based on global heat balances, the model does not yield, however, the internal temperature distribution in the electrolyser whose determination would require the employment of specific internal mass and heat balance sub-models for the electrolyser, taking into account hydrodynamic and mass-transport phenomena. The model proposed in this paper would nevertheless be useful in the case of internal temperature-profile studies/simulations as at least a preliminary guide for a more comprehensive analysis.

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Appendix
Derivation of Equation 4 from a differential thermal balance

Consider an infinitesimally short section of the recycle line of length dx, with heat transport area $dA_T = 2r\pi dx$, where r is the active radius (i.e., it includes the thickness of the lagging). Then, the differential thermal balance may be written as

$$Q_R\rho cT - Q_R\rho c\left(T + \frac{dT}{dx}dx\right) - 2U_T r\pi(T - T_A)\,dx = 0 \quad (A.1)$$

Upon simplification, Equation (A.2):

$$\frac{dT}{dx} = -\frac{2U_T r\pi}{Q_R\rho c}(T - T_A) \quad (A.2)$$

is integrated to

$$T_R = T_A + (T_2 - T_A)\exp\left(-\frac{U_T A_T}{Q_R\rho c}\right) \quad (4)$$

since at $x = 0$, $T = T_2$; at $x = L_R$, $T = T_R$ and $A_T = 2r\pi L_R$.